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Exponentiated exponential models for survival data

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Abstract

The Exponentiated Exponential (EE) model serves as an alternative to Exponential, Weibull and Gamma models. It is observed that EE model has been used in the analysis of complete life time data. In this paper an attempt has been made to study the modeling of censored survival data and the results are compared with other models. Log Likelihood ratio statistic and Cox-Snell residuals are used for the comparisons. The EE model performs better than Exponential and Weibull models. We also fitted Log-logistic model and compared with other models based on Baysian information criterion (BIC) and an information criterion (AIC). The Log-logistic model also performs better than the above models in situations when the censoring is at low level.

Keywords: EE model; hazard function; life time data; survival function; Weibull model

Introduction

The analysis of survival data provides a useful way of expressing the experience of a group of patients in the form of survivor function and hazard rate. The hazard rate measures the instantaneous probability of dying at a given time, conditional on patient having survival thus far (Collett, 1994). Any model which attempts to represent realistically the patient's survival experience may be judged for its suitability by the closeness of the theoretical survival function derived under the model to the survival function derived empirically from the data.

Models fitted to survival data may involve parametric or non-parametric forms for the hazard function. This depends on whether this form is defined (up to a small number of unknown parameters) as that of a known model, or whether it is completely undefined. This paper considered both parametric and non-parametric methods.

The EE model is very similar in shape to the Weibull model but found to be more suitable for the analysis of survival data. EE model has been applied to real data and empirical comparisons are presented for modeling survival data.

Exponentiated Exponential Model (EEM)

The two parameter model, EEM is defined as a particular case of Gompertz-Verhulst distribution function (Ahuja & Nash, 1967). The EEM has been discussed by (Gupta & Kundu, 1999). The cumulative distribution function of EEM (Gupta & Kundu, 1999, 2000, 2001 and 2003) is defined by

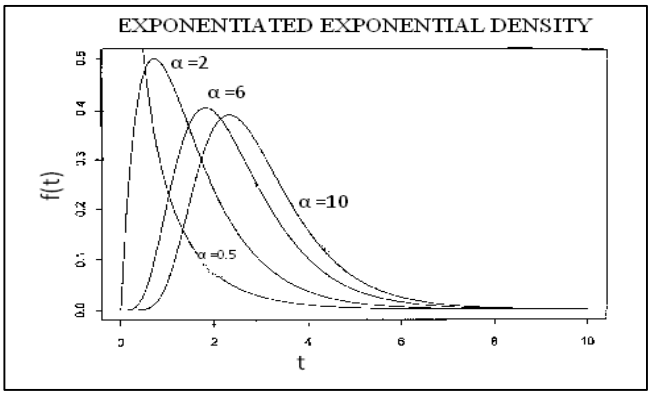
$$F_{EE}(t, \alpha, \lambda) = \left(1 - e^{-\lambda t}\right)^{\alpha}; \alpha, \lambda, t>0 \qquad (1)$$

and density function is defined by

$$f_{EE}(t, \alpha, \lambda) = \alpha\lambda\left(1 - e^{-\lambda t}\right)^{\alpha-1} e^{-\lambda t}; \alpha, \lambda, t > 0, \qquad (2)$$

Where α and λ are respectively shape and scale parameters. If α = 1, it results in the exponential family and it also represents the Gamma and Weibull model (Gupta & Kundu, 2001). For large values of α, the model converges to a symmetric form. The EE density function varies significantly depending of the shape parameter with λ = 1 (Fig.1).

Fig. 1. Density function of the EE model: λ = 1



Survival function is

$$S_{EE}(t, \alpha,\lambda) = P(T > t) = 1-\left(1-e^{-\lambda t}\right)^{\alpha}; \alpha, \lambda, t>0, \qquad (3)$$

the p^{th} percentile of the survival distribution is defined (Collett, 1994) as follows

$$t(p) = 1-\left(\frac{p}{100}\right), \qquad (4)$$

i.e.,

$$t_{EE}(p) = \frac{1}{\lambda}\ln\left\{\frac{1}{1-\left(\frac{p}{100}\right)^{\frac{1}{\alpha}}}\right\}, \qquad (5)$$

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the hazard function

$$h_{EE}(t,\alpha,\lambda)=\frac{\alpha\lambda\left(1-e^{-\lambda t}\right)^{\alpha-1}e^{-\lambda t}}{1-\left(1-e^{-\lambda t}\right)^{\alpha}};\ \alpha,\ \lambda,\ t>0 \quad (6)$$

and cumulative hazard is

$$H_{EE}(t,\ \alpha,\ \lambda)=-\log S(t);\qquad \alpha,\ \lambda,\ t>0. \quad (7)$$

Table 1. Hazard functions for various models

Parameter	MODELS			
	Exponential	Weibull	EE	Log-logistic
α = 1	Constant	Constant	Constant	Starts at $\lambda^{1/\alpha}$ and then declines monotonically
α > 1	Constant	Increasing from 0 to ∞	Increasing from 0 to λ	Increases to a peak at $t=(\alpha-1)^{1/\alpha}/\lambda^{1/\alpha}$ and then declines
α < 1	Constant	Decreasing from ∞ to 0	Decreasing from ∞ to λ	Starts at infinity and then declines

It is unimodal density function and for fixed scale parameter as the shape parameter increases, it is becoming more and more symmetric. For any λ, the hazard function is non-decreasing function if α>1, and it is a non-increasing function if α<1. For α=1, it is constant (Fig.2).

The notations in this paper defined by (Lee & Wang, 2003; Lawless, 2003)

Exponential Model (EM)

$$f_E(t)=\lambda e^{-\lambda t};\ \lambda,\ t>0, \quad (8)$$

using (4) the *p*th percentile of the EM of survival distribution is

$$t_E(p)=\frac{1}{\lambda}\ln\left(\frac{100}{100-p}\right). \quad (9)$$

Weibull Model (WM)

$$f_W(t)=\alpha\lambda(\lambda t)^{\alpha-1}e^{-(\lambda t)^{\alpha}};\ \alpha,\ \lambda,\ t>0, \quad (10)$$

using (4) the *p*th percentile of the WM of survival distribution is

$$t_W(p)=\frac{1}{\lambda}\left\{\ln\left(\frac{100}{100-p}\right)\right\}^{\frac{1}{\alpha}}, \quad (11)$$

Log-logistic Model (LLM)

$$f_{LL}(t)=\frac{\alpha\lambda t^{\alpha-1}}{\left(1+\lambda t^{\alpha}\right)^2};\ \alpha,\ \lambda,\ t>0, \quad (12)$$

using (4) the *p*th percentile of the LLM of survival distribution is

$$t_{LL}(p)=\left\{\frac{p}{\lambda(100-p)}\right\}^{\frac{1}{\alpha}}, \quad (13)$$

where α and λ are respectively shape and scale parameters of the models and the comparison of the three different hazard functions are given in Table 1.

Methods and materials

Maximum Likelihood Estimation

When we observe the survival data, it might include censored survival times. For this observed survival data, parametric models can be fitted by the method of maximum likelihood. We consider, the observed survival times of *"n"* individuals, that there are uncensored then, the probability density function of the random variable *(T)* associated with survival time is $f_T(t)$ and the likelihood of the observations $t_1,t_2,\ldots,t_n$ is defined by (Collett, 1994; Lawless, 2003; Aitkin, 1980; Clayton, 1983)

$$f_T(t)=\prod_{i=1}^{n} f(t_i). \quad (14)$$

We now consider where the survival data has censored survival time's *r* out of *n* individuals have the event at times $t_1,t_1,\ldots,t_1$ and the remaining *(n-r)* individuals $t_1^*,t_2^*,\ldots,t_{n-r}^*$ are right-censored. The overall likelihood function of the *r* events can be written as

$$f_{T(U)}(t)=\prod_{j=1}^{r} f_{T(U)}(t_j). \quad (15)$$

If we know that the life time of an individual is at least t^* (survival time is censored) and the probability of this event is $P(T\geq t^*)$, which is $S_{T(C)}(t^*)$. Therefore, the overall likelihood function of the *n* individuals defined by

$$L=\prod_{j=1}^{r} f_{T(U)}(t_j)\prod_{l=1}^{n-r} S_{T(C)}(t_l^*). \quad (16)$$

Conveniently, the *n* pairs of observations for *i*th individual is $(t_i,\delta_i;i=1,2,\ldots,n)$, where δ1 is an indicator variable

$$\delta_i=\begin{cases}1 & \text{if uncensored}\\ 0 & \text{if censored}\end{cases}$$

The total likelihood function can be written in the following way

$$L=\prod_{i=1}^{n}\left\{f_{T(U)}(t_i)\right\}^{\delta_i}\left\{S_{T(C)}(t_i)\right\}^{1-\delta_i}. \quad (17)$$

The above likelihood function (17) is equivalent to equation (16) (Collett, 1994; Lawless, 2003; Miller, 1981). This can be maximized with respect to the unknown parameters in to the probability density function and survival function.

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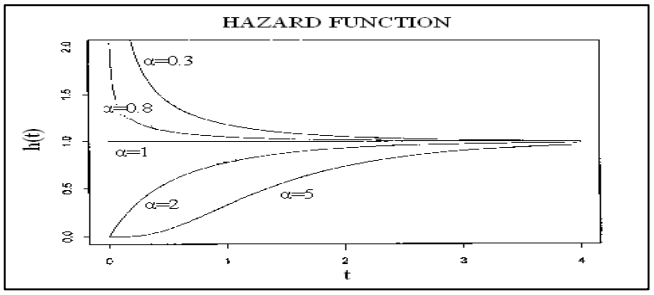
We now consider, single sample survival times of *n* individuals with $t_1, t_2, \ldots, t_n$ are assuming EEM. Then the data of the given actual death times of *r* individual and the remaining *(n-r)* survival times are right censored. As a special case fitting of the two parameter EEM (2) was used in expression (17). The log likelihood function can be written as

$$LL = \sum_{i=1}^{n} \delta_i \ln\left\{f_{T(U)}(t_i)\right\} + \sum_{i=1}^{n} (1-\delta_i) \ln\left\{S_{T(C)}(t_i)\right\} \quad (18)$$

To estimate the value of λ and α for which the LL function is maximum, differentiating with respect to λ and α, we have

Fig.2. Hazard functions of the EE model: λ =1



$$\frac{dLL}{d\lambda} = \sum_{i=1}^{n} \delta_i \left[\frac{1}{\lambda} - t_i + (\alpha-1)\frac{t_i e^{-\lambda t_i}}{\left(1-e^{-\lambda t_i}\right)}\right] - \alpha \sum_{i=1}^{n} (1-\delta_i) \frac{t_i e^{-\lambda t_i}\left(1-e^{-\lambda t_i}\right)^{\alpha-1}}{\left[1-\left(1-e^{-\lambda t_i}\right)^{\alpha}\right]}, \quad (19)$$

$$\frac{dLL}{d\alpha} = \sum_{i=1}^{n} \delta_i \left[\frac{1}{\alpha} + \ln\left(1-e^{-\lambda t_i}\right)\right] - \sum_{i=1}^{n} (1-\delta_i) \frac{\left(1-e^{-\lambda t_i}\right)^{\alpha} \ln\left(1-e^{-\lambda t_i}\right)}{\left[1-\left(1-e^{-\lambda t_i}\right)^{\alpha}\right]}. \quad (20)$$

Since the data contain *r* deaths, $\Sigma_i\delta_i$=r and the remaining are right censored, $\Sigma_i(1-\delta_i)$= n-r then the above log likelihood function (19) and (20) rewritten as

$$\frac{dLL}{d\lambda} = \frac{r}{\lambda} + (\alpha-1)\sum_{j=1}^{r} \frac{t_j e^{-\lambda t_j}}{\left(1-e^{-\lambda t_j}\right)} - \sum_{j=1}^{r} t_j - \alpha \sum_{l=1}^{n-r} \frac{t_l e^{-\lambda t_l}\left(1-e^{-\lambda t_l}\right)^{\alpha-1}}{\left[1-\left(1-e^{-\lambda t_l}\right)^{\alpha}\right]}, \quad (21)$$

$$\frac{dLL}{d\alpha} = \frac{r}{\alpha} + \sum_{j=1}^{r} \ln\left(1-e^{-\lambda t_j}\right) - \sum_{l=1}^{n-r} \frac{\left(1-e^{-\lambda t_l}\right)^{\alpha} \ln\left(1-e^{-\lambda t_l}\right)}{\left[1-\left(1-e^{-\lambda t_l}\right)^{\alpha}\right]}, \quad (22)$$

$$\frac{r}{\lambda} + (\alpha-1)\sum_{j=1}^{r} \frac{t_j e^{-\lambda t_j}}{\left(1-e^{-\lambda t_j}\right)} - \sum_{j=1}^{r} t_j - \alpha \sum_{l=1}^{n-r} \frac{t_l e^{-\lambda t_l}\left(1-e^{-\lambda t_l}\right)^{\alpha-1}}{\left[1-\left(1-e^{-\lambda t_l}\right)^{\alpha}\right]} = 0, \quad (23)$$

$$\frac{r}{\alpha} + \sum_{j=1}^{r} \ln\left(1-e^{-\lambda t_j}\right) - \sum_{l=1}^{n-r} \frac{\left(1-e^{-\lambda t_l}\right)^{\alpha} \ln\left(1-e^{-\lambda t_l}\right)}{\left[1-\left(1-e^{-\lambda t_l}\right)^{\alpha}\right]} = 0. \quad (24)$$

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Fig. 3(A). Plot for the fitted Survivor functions of Exponential, Weibull, EE and LL models and Kaplan-Meier for IUD data

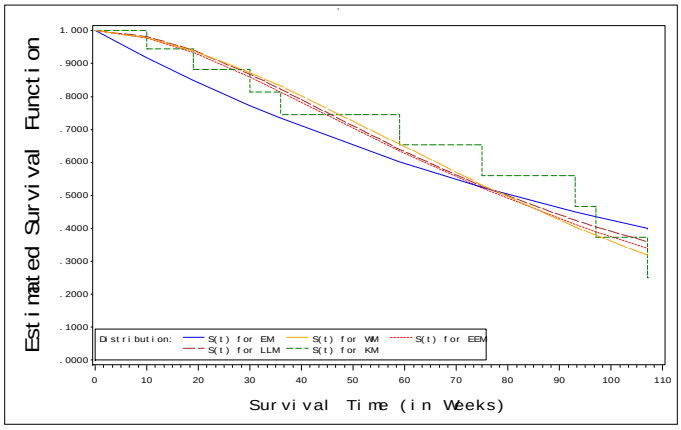

Fig. 3(B). Plot for the fitted Survivor functions of Exponential,Weibull, EE and LL models and Kaplan-Meier for multiple myeloma data

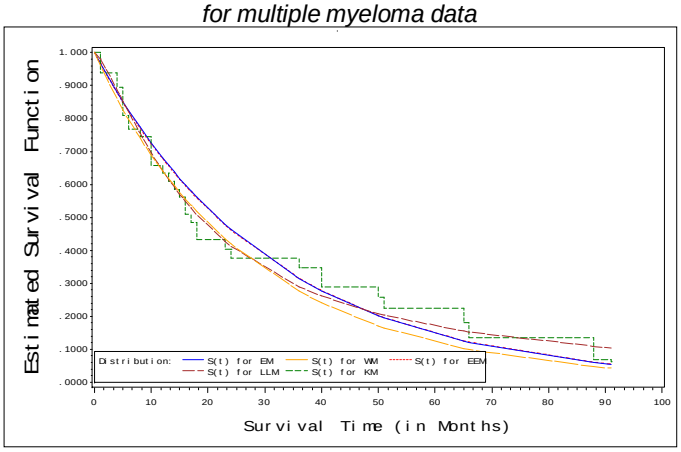

Fig.3(C). Plot for the fitted Survivor function of Exponential, Weibull, EE and LL models and Kaplan-Meier for spinal TB data.

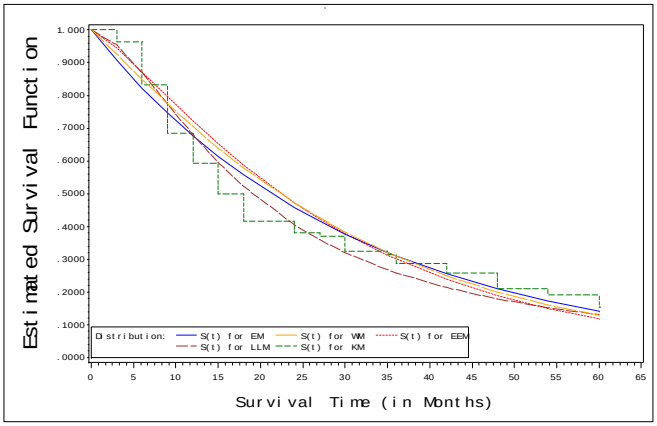

Fig. 3(D). Plot for the fitted hazard functions of Exponential, Weibull, EE and LL models for IUD data

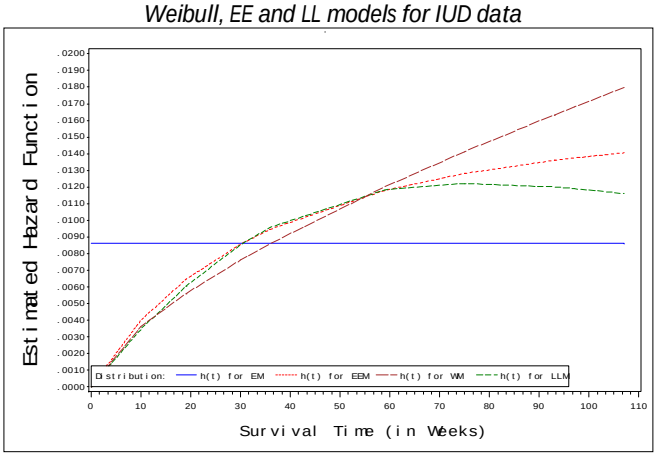

Fig. 3(E). Plot for the fitted hazard functions of Exponential, Weibull, EE and LL models for multiple myeloma data

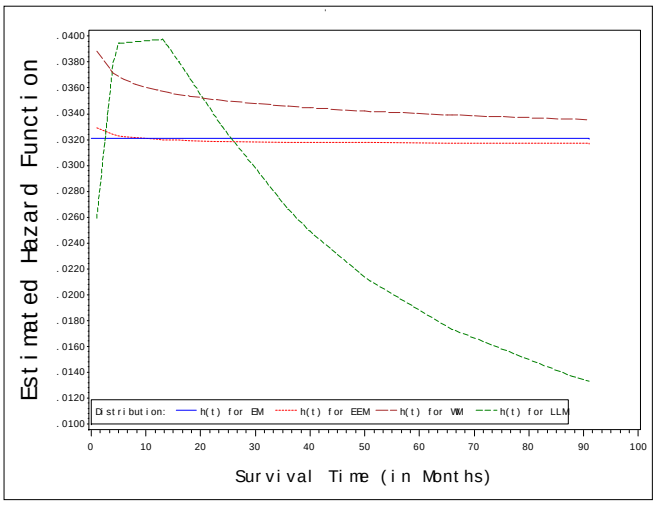

Fig. 3(F). Plot for the fitted hazard functions of Exponential, Weibull, EE and LL models for spinal TB data

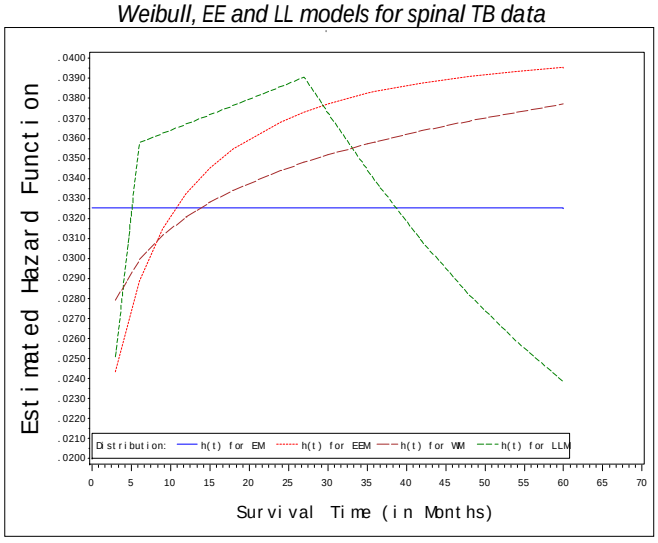

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Fig.3(G).Cox-Snell residual plot for the fitted Exponential, Weibull, EE and LL models for IUD data

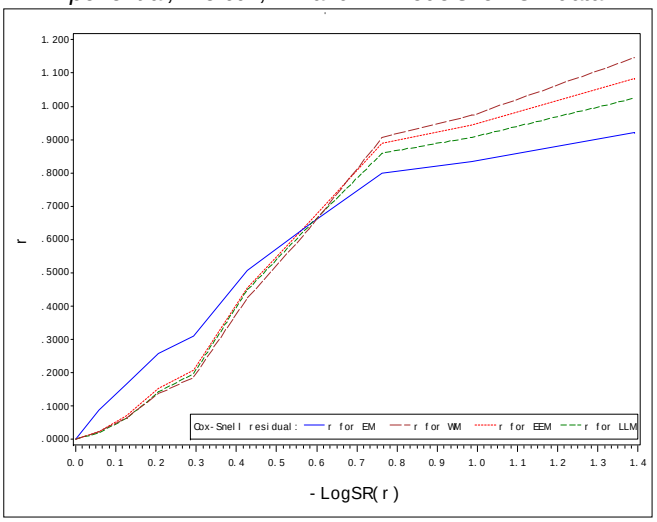


Fig. 3(H). Cox-Snell residual plot for the fitted Exponential, Weibull,EE and LL models for multiple myeloma data.

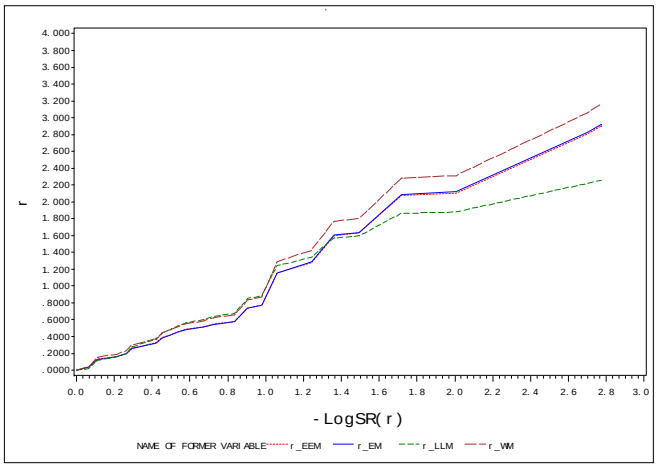
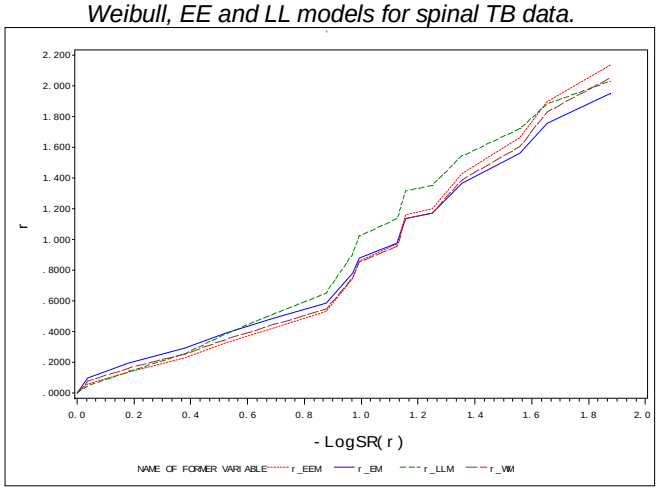


Fig. 3(I). Cox-Snell residual plot for the fitted Exponential, Weibull, EE and LL models for spinal TB data.



Here $t_1,t_2,\ldots,t_r$ *(j = 1,2,…,r)* and $t_1^*,t_2^*,\ldots,t_{n-r}^*$ *(l = 1,2,…,n-r)* are respectively uncensored and censored survival times of *n* individuals. Equating the derivative of equation (21) and (22) to zero, we get (23) and (24).

This is the non-linear equation in λ and α which can only be solved using numerical methods, such as the Newton-Raphson method.

Databases

We have considered three survival data sets for the empirical comparisons and this section gives a brief description of the data sets.

IUD and pregnancy data (Collett, 1994)

Data consist of 18 women, all of whom were aged between 18 and 35 years and who had experienced two previous pregnancies. In this problem, the time origin corresponds to the first day in which a woman uses the intrauterine device (IUD) and the end-point is discontinuation because of bleeding problems. Some women in the study ceased using the IUD because of the desire for pregnancy, or because they had no further need for a contraceptive, while others were simply lost to follow -up. Out of 18, 9 (50%) observations were censored. The survival times were in weeks.

Multiple myeloma data (Collett, 1994)

The data relate to 48 patients with multiple myeloma, all of whom were aged between 50 and 80 years. Out of these patients, 12 (25%) had not died by the time that the study was completed, and so these individuals contribute right-censored survival times. The survival times were recorded in months.

Spinal tuberculosis Data (ICMR/MRC, 1989)

This data relates to 108 patients. A sub group of 304 patients admitted to a clinical trial with a diagnosis of tuberculosis involving thoracic or lumbar spine allocated to one of the three treatment series. Out of the 108 patients response time considered, 17 (16%) response times were censored. The response times were recorded in months.

Results and discussion

The parameter estimates for the Exponential, Weibull, Exponentiated Exponential and Log-logistic are given in Table 2 along the 2.5th and 97.5th percentiles. The SAS package has been used to fit the models (Appendix A gives the programme code for EEM).

From Table 2 we observe that the deviance of the exponential is slightly higher than the EE and Weibull models; however the difference is not significant. Further all the three models give similar median survival time. However the percentiles vary considerably. As the percentage of censoring decreases, the EEM gives a narrow confidence interval than the other two models. For the spinal tuberculosis data the EEM gave a shorter 95% interval than WM and EM. The Cox-Snell residuals method also indicates that EEM is better than Weibull and Exponential.

The estimated survivor and hazard functions of Exponential, Weibull, EE and LL models along with Kaplan-Meier are shown in Tables 3(A)- 3(C) and graphical representations of the survival function, hazard

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Table 2. Parameter estimates for the models for the three data sets.

Parameters & Methods		λ	α	-2LL	Percentiles			BIC	AIC
Models					2.5th	Median	97.5th		
IUD Data	EM	0.0086	-	103.60	2.94	80.60	428.94	-53.25	-53.80
	WM	0.01014	1.676	100.70	11.00	79.25	214.88	-53.24	-54.35
	EEM	0.015661	1.9956	101.08	10.94	78.29	279.26	-53.43	-54.54
	LLM	0.00021026	1.93610842	101.74	11.95	79.30	526.12	-53.76	-54.87
Multiple myeloma data	EM	0.0320855	-	319.64	0.79	21.60	114.97	-161.76	-161.82
	WM	0.0360394	0.9674923	319.56	0.62	19.00	106.95	-163.65	-163.78
	EEM	0.031702	0.9851	319.62	0.75	21.54	115.89	-163.68	-163.81
	LLM	0.01958215	1.348059	319.74	1.22	18.50	280.14	-163.74	-163.87
Spinal TB Data	EM	0.0325232	-	805.50	0.78	21.31	113.42	-405.09	-404.75
	WM	0.03208	1.1005	804.28	1.10	22.34	102.07	-406.82	-406.14
	EEM	0.04019	1.334	801.68	1.62	22.48	98.88	-405.52	-404.84
	LLM	0.00785436	1.646090535	790.68	2.05	19.00	175.91	-400.02	-399.34

function and Cox-Snell residuals are presented in Fig. 3(A) - 3(I).

The results of fitting of the LLM for the three data bases are presented in Table 2. Based on -2LL statistics, BIC and AIC (Lee & Wang, 2003) we observed that LLM is providing a better fit than the other models for the spinal TB data when the censoring is low. In high censoring situation all models perform in the same level.

unknown shape and scale parameters of life time for censored data. Using the maximum likelihood and SAS, the shape and scale parameters has been estimated.

Three data bases to illustrate the application of this model are presented. It is observed that the EEM seems to be more appropriate for the spinal TB data when compared to an Exponential and Weibull models. The performance of EEM improves considerably in low censored situation.The LLM is found to provide a better fit

Table 3(A). Estimated Survivor and hazard functions of EM, WM, EEM, LLM and Kaplan-Meier for IUD data

Survival time		Survival Function				Hazard Function			
	$\hat{S}(t)\_EM$	$\hat{S}(t)\_WM$	$\hat{S}(t)\_EEM$	$\hat{S}(t)\_LLM$	$\hat{S}(t)\_KM$	$\hat{h}(t)\_EM$	$\hat{h}(t)\_EEM$	$\hat{h}(t)\_WM$	$\hat{h}(t)\_LLM$
0-	1.00000	1.00000	1.00000	1.00000	1.0000	0.00860	0.00000	0.00000	0.00000
10-	0.91759	0.97865	0.97881	0.98217	0.9444	0.00860	0.00399	0.00362	0.00345
19-	0.84925	0.93867	0.93336	0.94083	0.8815	0.00860	0.00644	0.00558	0.00603
30-	0.77260	0.87278	0.85885	0.86785	0.8137	0.00860	0.00856	0.00760	0.00853
36-	0.73374	0.83135	0.81359	0.82187	0.7459	0.00860	0.00946	0.00860	0.00958
59-	0.60206	0.65525	0.63549	0.63937	0.6526	0.00860	0.01180	0.01201	0.01183
75-	0.52466	0.53152	0.52167	0.52698	0.5594	0.00860	0.01281	0.01412	0.01221
93-	0.44942	0.40400	0.41111	0.42350	0.4662	0.00860	0.01360	0.01633	0.01200
97-	0.43422	0.37809	0.38923	0.40372	0.3729	0.00860	0.01374	0.01681	0.01190
107	0.39844	0.31775	0.33871	0.35895	0.2486	0.00860	0.01405	0.01796	0.01160

Conclusions

Gupta and Kundu (2003) discussed and found that, the EEM is better fit than the other models based on the some methods applied for complete life time data sets. In this paper we consider a special case of EEM with for spinal TB data when compared to other models in the low censoring situation. Further studies are needed to validate this conclusion under different censoring patterns.

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Table 3(B). Estimated Survivor and hazard functions of EM, WM, EEM, LLM and Kaplan-Meier for multiple myeloma data.

Survival time		Survival Function				Hazard Function			
	$\hat{S}(t)$ _ EM	$\hat{S}(t)$ _ WI	$\hat{S}(t)$ _ EEM	$\hat{S}(t)$ _ LLM	$\hat{S}(t)$ _ KM	$\hat{h}(t)$ _ EM	$\hat{h}(t)$ _ EEM	$\hat{h}(t)$ _ WM	$\hat{h}(t)$ _ LLM
0-	1.00000	1.00000	1.00000	1.00000	1.0000	0.03209	0.00000	0.00000	0.00000
1-	0.96842	0.96064	0.96714	0.98079	0.9375	0.03209	0.03294	0.03885	0.02589
4-	0.87955	0.85768	0.87707	0.88739	0.8949	0.03209	0.03238	0.03713	0.03795
5-	0.85178	0.82653	0.84916	0.85365	0.8097	0.03209	0.0323	0.03687	0.03946
6-	0.82488	0.7967	0.8222	0.82021	0.7670	0.03209	0.03223	0.03665	0.04040
8-	0.77361	0.74066	0.77094	0.75583	0.7451	0.03209	0.03214	0.03631	0.04115
10-	0.72553	0.68897	0.72299	0.69617	0.6575	0.03209	0.03208	0.03604	0.04096
12-	0.68043	0.6412	0.6781	0.64183	0.6340	0.03209	0.03203	0.03583	0.04024
13-	0.65895	0.61866	0.65673	0.61667	0.6096	0.03209	0.032	0.03574	0.03975
14-	0.63814	0.59696	0.63605	0.59279	0.5852	0.03209	0.03199	0.03565	0.03921
15-	0.61799	0.57608	0.61604	0.57017	0.5608	0.03209	0.03197	0.03557	0.03863
16-	0.59848	0.55597	0.59666	0.54873	0.5098	0.03209	0.03195	0.0355	0.03802
17-	0.57958	0.5366	0.5779	0.52842	0.4844	0.03209	0.03194	0.03543	0.03740
18-	0.56128	0.51794	0.55974	0.50919	0.4334	0.03209	0.03193	0.03536	0.03676
23-	0.47809	0.43432	0.47722	0.42710	0.4045	0.03209	0.03187	0.03508	0.03358
24-	0.46299	0.41936	0.46225	0.41312	0.3756	0.03209	0.03187	0.03503	0.03296
36-	0.31503	0.27624	0.3155	0.28953	0.3467	0.03209	0.0318	0.03457	0.02660
40-	0.27709	0.24062	0.27782	0.26121	0.2889	0.03209	0.03178	0.03446	0.02490
50-	0.20104	0.17071	0.20221	0.20743	0.2568	0.03209	0.03176	0.03421	0.02137
51-	0.19469	0.16497	0.19589	0.20307	0.2247	0.03209	0.03176	0.03418	0.02106
65-	0.12424	0.10242	0.1256	0.15523	0.1798	0.03209	0.03173	0.03392	0.01752
66-	0.12031	0.09901	0.12168	0.15255	0.1348	0.03209	0.03173	0.0339	0.01731
88-	0.0594	0.04714	0.06055	0.10885	0.0674	0.03209	0.03172	0.03358	0.01365
91	0.05395	0.04262	0.05505	0.10454	0.0625	0.03209	0.03172	0.03355	0.01327

Table 3(C). Estimated Survivor and hazard functions of EM, WM, EEM, LLM and Kaplan-Meier for spinal TB data

0-	1.00000	1.00000	1.00000	1.00000	1.0000	0.03252	0.00000	0.00000	0.00000
3-	0.90704	0.92676	0.94507	0.95427	0.9630	0.03252	0.02432	0.0279	0.02509
6-	0.82272	0.8495	0.87192	0.86958	0.8333	0.03252	0.02888	0.02992	0.03578
9-	0.74624	0.77505	0.79619	0.77379	0.6852	0.03252	0.03149	0.03116	0.04137
12-	0.67687	0.70487	0.7224	0.68055	0.5926	0.03252	0.03324	0.03207	0.04382
15-	0.61395	0.63948	0.65253	0.59604	0.5000	0.03252	0.03451	0.0328	0.04433
18-	0.55687	0.57901	0.58748	0.52220	0.4167	0.03252	0.03547	0.03341	0.04369
24-	0.45815	0.47239	0.4728	0.40500	0.3796	0.03252	0.03682	0.03439	0.04081
27-	0.41556	0.42582	0.42304	0.35926	0.3704	0.03252	0.03731	0.0348	0.03906
30-	0.37693	0.38339	0.378	0.32039	0.3241	0.03252	0.03772	0.03517	0.03729
35-	0.32036	0.32112	0.3126	0.26781	0.3148	0.03252	0.03825	0.03572	0.03444
36-	0.31011	0.30984	0.30085	0.25882	0.2870	0.03252	0.03834	0.03582	0.03389
42-	0.25513	0.24949	0.23869	0.21318	0.2583	0.03252	0.03879	0.03638	0.03084
48-	0.2099	0.20026	0.18893	0.17863	0.2105	0.03252	0.03912	0.03687	0.02817
54-	0.17269	0.16031	0.14929	0.15193	0.1914	0.03252	0.03937	0.03731	0.02585
60	0.14208	0.128	0.11782	0.13091	0.1531	0.03252	0.03955	0.03771	0.02384

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Appendix A

α and λ for EE model using (3.10) and (3.11) we written the SAS programme.

```
data pv3;
input t s $20.;
datalines;
(Enter the values of survival time t and status s);
run;
proc print data = pv3;
run;
proc freq data=pv3;
table s/out=sfreq;
run;
data _null_;
set sfreq;
if s="censored" then do;
call symputx("cen",count);
end;
if s="uncensord" then do;
call symputx("uncen",count);
end;
run;
proc sort data=pv3;
by s;
run;
proc transpose data=pv3 (where=(s="uncensord"))
out=pv3unc prefix=UNC;
run;
proc transpose data=pv3 (where=(s="censored")) out=pv3c
prefix=C;
run;
data pv33;
merge pv3unc pv3c;
by _name_;
array unc{&uncen} unc:;     /*equa-part--->1*/
array cen{&cen} c:;  /*equa-part--->1*/
array lcen{&cen};     /*equa-part--->1*/
array lunc{&uncen}; /*equa-part--->1*/
array loglcen{&cen};
array loglunc{&uncen};
array logunc{&uncen};
do i= (specify the range);
do j= (specify the range);
LSCEN=0;
scen=0;
sunc=0;
lsunc=0;
logs=0;
do k=1 to &cen;
lcen{k}=((((1-exp(-i*cen{k}))**(j-1))*cen{k}*exp(-i*cen{k}))/(1-
(1-exp(-i*cen{k}))**j));    /*equa1-part4*/
loglcen{k}=((((1-exp(-i*cen{k}))**j)*log(1-exp(-i*cen{k})))/(1-
(1-exp(-i*cen{k}))**j));   /*equa2-part3*/
scen=scen+lcen{k};
lscen=lscen+loglcen{k};
end;
do l=1 to &uncen;
lunc{l}=unc{l};              /*equa1-part3*/
loglunc{l}=((unc{l}*exp(-i*unc{l}))/(1-exp(-i*unc{l}))); /*equa1-
part2*/
logunc{l}=log(1-exp(-i*unc{l})); /*equa2-part2*/
sunc=sunc+lunc{l};
lsunc=lsunc+loglunc{l};
logs=logs+logunc{l};
end;
res=(&uncen./i) +((j-1)*lsunc)-sunc-(j*scen);/*---> eqn 1*/
res1=(&uncen./j)+ logs - lscen ;            /*---> eqn 2*/
if round(res,1.1111) = 0 or round(res1,1.11) = 0 then do;
abs=res;
abs1=res1;
output;
end;
end;
end;
run;
proc sort data=pv33 out=pv33_e1;
by res;
run;
proc sort data=pv33 out=pv33_e2;
by res1;
run;
```